Eur. Phys. J. C 46, 825–828 (2006)
Digital Object Identifier (DOI) 10.1140/epjc/s2006-02538-2

THE EUROPEAN PHYSICAL JOURNAL C

The Aharonov–Bohm effect in noncommutative quantum mechanics

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Received: 25 August 2005 / Revised version: 22 February 2006 /
Published online: 12 April 2006 –

Abstract. The Aharonov–Bohm effect in noncommutative (NC) quantum mechanics is studied. First, by introducing a shift for the magnetic vector potential we give the Schrödinger equations in the presence of a magnetic field on NC space and NC phase space, respectively. Then, by solving the Schrödinger equations, we obtain the Aharonov–Bohm phase on NC space and NC phase space, respectively.

PACS. 11.10.Nx; 03.65.-w

1 Introduction

Recently, there has been much interest in the study of physics on noncommutative (NC) space [1–6], not only because the NC space is necessary when one studies the low-energy effective theory of a D-brane with a B-field background, but also because in the very tiny string scale or in a very high energy situation the effects of noncommutativity of both space–space and momentum–momentum may appear. There are many papers devoted to the study of various aspects of quantum mechanics [7–34] on noncommutative space with the usual (commutative) time coordinate.

In the usual n-dimensional commutative space, the coordinates and momenta in quantum mechanics have the following commutation relations:

$$\begin{aligned} &[x_i, x_j] = 0\,, \\ &[p_i, p_j] = 0\,, \qquad i,j = 1,2,\ldots,n \\ &[x_i, p_j] = \mathrm{i}\hbar\delta_{ij}\,. \end{aligned} \tag{1}$$

At very tiny scales, say string scale, not only space–momentum does not commute, but also space–space may not commute any more. Therefore, the NC space is a space where coordinate and momentum operators satisfy the following commutation relations:

$$\begin{aligned} &[\hat{x}_i, \hat{x}_j] = \mathrm{i}\Theta_{ij}\,, \\ &[\hat{p}_i, \hat{p}_j] = 0\,, \\ &[\hat{x}_i, \hat{p}_j] = \mathrm{i}\hbar\delta_{ij}\,, \end{aligned} \tag{2}$$

where $\hat{x}_i$ and $\hat{p}_i$ are the coordinate and momentum operators on NC space. Li et al. [34] showed that $\hat{p}_i = p_i$ and $\hat{x}_i$ have the following form:

$$\hat{x}_i = x_i - \frac{1}{2\hbar}\Theta_{ij}p_j\,, \qquad i,j = 1,2,\ldots,n\,. \tag{3}$$

The case of both space–space and momentum–momentum noncommuting [30, 34] is different from the case of only space–space noncommuting. Thus, the NC phase space is a space where the momentum operator in (2) satisfies the following commutation relations:

$$[\hat{p}_i, \hat{p}_j] = \mathrm{i}\bar{\Theta}_{ij}\,, \qquad i,j = 1,2,\ldots,n\,. \tag{4}$$

Here $\{\Theta_{ij}\}$ and $\{\bar{\Theta}_{ij}\}$ are totally antisymmetric matrices which represent the noncommutative property of the coordinate and momentum on noncommutative space and phase space, respectively, and play an analogous role to $\hbar$ in the usual quantum mechanics. On NC phase space the representations of $\hat{x}$ and $\hat{p}$ in terms of x and p were given in [34] as follows:

$$\begin{aligned} &\hat{x}_i = \alpha x_i - \frac{1}{2\hbar\alpha}\Theta_{ij}p_j\,, \\ &\hat{p}_i = \alpha p_i + \frac{1}{2\hbar\alpha}\bar{\Theta}_{ij}x_j\,. \qquad i,j = 1,2,\ldots,n \end{aligned} \tag{5}$$

Here α is a scaling constant related to the noncommutativity of phase space. When $\bar{\Theta} = 0$, it leads to $\alpha = 1$ [34]; the NC phase space returns to the NC space, which is extensively studied in the literature, where space–space is noncommuting, while momentum–momentum is commuting.

Given the NC space or NC phase space, one should study its physical consequences. It appears that the most natural places to search for the noncommutativity effects are a simple quantum mechanics (QM) system. So far, many interesting topics in NCQM such as the hydrogen atom spectrum in

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an external magnetic field [12, 15] and the Aharonov–Bohm (AB) effect [14] in the presence of a magnetic field, as well as the Aharonov–Casher (AC) effects [16], have been studied extensively. The purpose of this paper is to carry out a further study of the AB effect on NC space and NC phase space, respectively, where the space–space noncommutativity and both space–space and momentum–momentum noncommutativity could produce an additional phase difference.

This paper is organized as follows: in Sect. 2, we study the AB effect on NC space. First, the Schrödinger equation in the presence of a magnetic field is given, and the magnetic AB phase expression is derived. In two dimensions, our result agrees with the result of [14]. The general AB phase on NC space is also given in the presence of an electromagnetic field. In Sect. 3, we investigate the AB effect on NC phase space. By solving the Schrödinger equation in the presence of a magnetic field, the additional AB phase related to the momentum–momentum noncommutativity is obtained explicitly. Conclusions and some remarks are given in Sect. 4.

2 The Aharonov–Bohm effect on NC space

Let $H(x,p)$ be the Hamiltonian operator of the usual quantum system; then the static Schrödinger equation on NC space is usually written as

$$H(x,p) * \psi = E\psi\,, \tag{6}$$

where the Moyal–Weyl (or star) product between two functions is defined by

$$\begin{aligned}(f*g)(x) &= \mathrm{e}^{(\mathrm{i}/2)\Theta_{ij}\partial_{x_i}\partial_{x_j}} f(x_i)g(x_j)\\ &= f(x)g(x) + \frac{\mathrm{i}}{2}\Theta_{ij}\partial_i f \partial_j g\big|_{x_i=x_j}\,;\end{aligned} \tag{7}$$

here $f(x)$ and $g(x)$ are two arbitrary functions. On NC space the star product can be replaced by a Bopp's shift [4], i.e. the star product can be changed into the ordinary product by replacing $H(x,p)$ with $H(\hat{x},\hat{p})$. Thus, the Schrödinger equation can be written as

$$H(\hat{x}_i,\hat{p}_i)\psi = H\left(x_i - \frac{1}{2\hbar}\Theta_{ij}p_j, p_i\right)\psi = E\psi\,. \tag{8}$$

Here x_i and p_i are coordinate and momentum operators in the usual quantum mechanics. Thus, (8) is actually defined on commutative space, and the noncommutative effects can be evaluated through the Θ-related term. Note that the Θ term always can be treated as a perturbation in QM, since $\Theta_{ij} \ll 1$.

When a magnetic field is involved, the Schrödinger equation (6) becomes

$$H(x_i,p_i,A_i) * \psi = E\psi\,. \tag{9}$$

To replace the star product in (9) with the usual product, first we need to replace x_i and p_i with a Bopp's shift; then we also need to replace the vector potential A_i with a shift given as follows:

$$A_i \to A_i + \frac{1}{2}\Theta_{lj}p_l\partial_j A_i\,. \tag{10}$$

Thus, the Schrödinger equation (9) in the presence of a magnetic field becomes

$$H\left(x_i - \frac{1}{2\hbar}\Theta_{ij}p_j, p_i, A_i + \frac{1}{2}\Theta_{lj}p_l\partial_j A_i\right)\psi = E\psi\,. \tag{11}$$

We should emphasize that the Bopp's shift and the shift (10) are equivalent to the star product in the Schrödinger equation (9).

Now let us consider a particle of mass m and charge q moving in a magnetic field with magnetic potential A_i; then the Schrödinger equation is (we choose units of $\hbar = c = 1$)

$$\frac{1}{2m}\left(p_i - qA_i - \frac{1}{2}q\Theta_{lj}p_l\partial_j A_i\right)^2\psi = E\psi\,. \tag{12}$$

In an analogous way as in the usual quantum mechanics, the solution of (12) reads

$$\psi = \psi_0 \exp\left[\mathrm{i}q\int_{x_0}^{x}\left(A_i + \frac{1}{2}\Theta_{lj}p_l\partial_j A_i\right)\mathrm{d}x_i\right]\,, \tag{13}$$

where ψ_0 is the solution of (12) when $A_i = 0$. The phase term of (13) is the so-called AB phase. If we consider a charged particle to pass a double slit, then the integral runs from the source x_0 through one of the two slits to the screen x; the coherent pattern will depend on the phase difference of the two paths. Thus, the total phase shift for the AB effect is

$$\begin{aligned}\Delta\Phi_{\mathrm{AB}} &= \delta\Phi_0 + \delta\Phi_\theta^{\mathrm{NC}}\\ &= \mathrm{i}q\oint A_i\mathrm{d}x_i + \frac{\mathrm{i}q}{2}\oint\Theta_{lj}(mv_l + qA_l)\partial_j A_i\mathrm{d}x_i\,,\end{aligned} \tag{14}$$

where the relation $mv_l = p_l - qA_l + O(\Theta)$ has been applied[1] and we omitted the second-order terms of Θ; the first term is the AB phase in the usual quantum mechanics, the second term is the correction to the usual AB phase due to space–space noncommutativity; the line integral runs from the source through one of the two slits to the screen and returns to the source through the other slit.

In three-dimensional NC space, i.e. $i,j = 1,2,3$, we can define a vector $\theta = (\theta_1,\theta_2,\theta_3)$ with θ_i satisfying $\Theta_{ij} = \epsilon_{ijk}\theta_k$, or $\theta_i = \frac{1}{2}\epsilon_{ijk}\Theta_{jk}$. Then the second and third terms in (14) have the form

$$\begin{aligned}\frac{\mathrm{i}}{2}q\oint\Theta_{lj}mv_l\partial_j A_i\mathrm{d}x_i &= \frac{\mathrm{i}q}{2}\oint\epsilon_{lmk}\theta_k mv_l\partial_j A_i\mathrm{d}x_i\\ &= \frac{\mathrm{i}q}{2}\oint m\theta\cdot(\mathbf{v}\times\nabla A_i)\mathrm{d}x_i\end{aligned} \tag{15}$$

and

$$\begin{aligned}\frac{\mathrm{i}}{2}q^2\oint\Theta_{lj}A_l\partial_j A_i\mathrm{d}x_i &= \frac{\mathrm{i}}{2}q^2\oint\epsilon_{lmk}\theta_k A_l\partial_j A_i\mathrm{d}x_i\\ &= \frac{\mathrm{i}}{2}q^2\oint\theta\cdot(\mathbf{A}\times\nabla A_i)\mathrm{d}x_i\,.\end{aligned} \tag{16}$$

[1] By (12), one writes the velocity operator on NC space as $v_l = \partial H/\partial p_l = (1/m)(p_l - qA_l - \frac{1}{2}q\Theta_{ij}p_i\partial_j A_l - \frac{1}{2}q\Theta_{lj}(p_i - qA_i)\partial_j A_i + O(\Theta^2)) = (1/m)(p_l - qA_l + O(\Theta))$.

Using (15) and (16), we can write the AB phase as

$$\Delta\Phi_{\mathrm{AB}} = \mathrm{i}q\oint A_i \mathrm{d}x_i + \frac{\mathrm{i}}{2}q\oint \big[m\theta\cdot(\mathbf{v}\times\nabla A_i) + q\theta\cdot(\mathbf{A}\times\nabla A_i)\big]\mathrm{d}x_i\,. \tag{17}$$

In a two-dimensional NC plane ($i,j=1,2$), if we consider an electron ($q=-e$) moving in a magnetic field, then the vector θ defined above just has the third component θ_3 and $\Theta_{ij}=\theta_3\epsilon_{ij}$, $\epsilon_{12}=-\epsilon_{21}=1, \epsilon_{11}=\epsilon_{22}=0$; then we have

$$\Delta\Phi_{\mathrm{AB}} = -\mathrm{i}e\oint A_i \mathrm{d}x_i - \frac{\mathrm{i}}{2}e\theta_3\oint \big[m(\mathbf{v}\times\nabla A_i)_3 - e(\mathbf{A}\times\nabla A_i)_3\big]\mathrm{d}x_i\,. \tag{18}$$

We should emphasize that, in a two-dimensional NC plane, our result (18) is exactly the same as in [14].

The AB phase (14) gives us a hint that when a charged particle moves in an electromagnetic field with four-dimensional potential A_μ, then the corresponding AB phase will have the following general expression:

$$\Delta\Phi_{\mathrm{AB}} = \mathrm{i}q\oint \left[A_\mu + \frac{1}{2}\Theta_{\alpha\beta}(mv_\alpha + qA_\alpha)\partial_\beta A_\mu\right]\mathrm{d}x^\mu\,. \tag{19}$$

The second term is the consequence of space–space noncommutativity.

3 The Aharonov–Bohm effect on NC phase space

The Bose–Einstein statistics in NCQM requires both space–space and momentum–momentum noncommutativity. Thus, we should also consider the momentum–momentum noncommutativity when we deal with physical problems. On NC phase space the noncommuting coordinates $\hat{x_i}$ and momentum $\hat{p_i}$ were given in (5). On NC phase space the star product in (7) becomes

$$\begin{aligned}(f*g)(x,p) &= \mathrm{e}^{(\mathrm{i}/2\alpha^2)\Theta_{ij}\partial_i^x\partial_j^x + (\mathrm{i}/2\alpha^2)\bar{\Theta}_{ij}\partial_i^p\partial_j^p} f(x,p)g(x,p)\\ &= f(x,p)g(x,p) + (\mathrm{i}/2\alpha^2)\Theta_{ij}\partial_i^x f\partial_j^x g\big|_{x_i=x_j}\\ &\quad + \frac{\mathrm{i}}{2\alpha^2}\bar{\Theta}_{ij}\partial_i^p f\partial_j^p g\big|_{p_i=p_j}\,.\end{aligned} \tag{20}$$

To replace the star product in Schrödinger (9) with the usual product, first we need to replace x_i and p_i with a generalized Bopp's shift as

$$\begin{aligned}x_i &\to x_i - \frac{1}{2\hbar\alpha^2}\Theta_{ij}p_j\,,\\ p_i &\to p_i + \frac{1}{2\hbar\alpha^2}\bar{\Theta}_{ij}x_j\,,\end{aligned} \tag{21}$$

and then we need to replace A_i with the generalized shift as

$$A_i \to \alpha A_i + \frac{1}{2\alpha}\Theta_{lj}p_l\partial_j A_i\,. \tag{22}$$

Thus, on NC phase space the Schrödinger equation (9) becomes

$$H\bigg(x_i - \frac{1}{2\hbar\alpha^2}\Theta_{ij}p_j, p_i + \frac{1}{2\hbar\alpha^2}\bar{\Theta}_{ij}x_j, A_i + \frac{1}{2\alpha^2}\Theta_{lj}p_l\partial_j A_i\bigg)\psi = E\psi\,. \tag{23}$$

One may note that (21) is different from (5). Now the other physical quantities may also be shifted; for example, mass may be replaced with $m\to m/\alpha^2$ and the electric charge q may be replaced with q/α.

Here, again, we consider a particle of mass m and electric charge q moving in a magnetic field. On NC phase space, the Hamiltonian has the form

$$\begin{aligned}\hat{H} &= \frac{1}{2m}\left(\alpha p_i + \frac{1}{2\alpha}\bar{\Theta}_{ij}x_j - q\left(\alpha A_i + \frac{1}{2\alpha}\Theta_{lj}p_l\partial_j A_i\right)\right)^2\\ &= \frac{1}{2m'}\left(p_i + \frac{1}{2\alpha^2}\bar{\Theta}_{ij}x_j - q\left(A_i + \frac{1}{2\alpha^2}\Theta_{lj}p_l\partial_j A_i\right)\right)^2,\end{aligned} \tag{24}$$

with $m'=m/\alpha^2$. Thus, the total phase shift for the AB effect including the contribution due to both space–space and momentum–momentum noncommutativity on three-dimensional NC phase space is[2]

$$\begin{aligned}\Delta\Phi_{\mathrm{AB}} &= \delta\Phi_{\mathrm{NCPS}}\\ &= \mathrm{i}q\oint A_i\mathrm{d}x_i + \frac{\mathrm{i}q}{2\alpha^2}\\ &\quad\times\oint\big[m'\theta\cdot(\mathbf{v}\times\nabla A_i) + q\theta\cdot(\mathbf{A}\times\nabla A_i)\big]\mathrm{d}x_i\\ &\quad - \frac{\mathrm{i}}{2\alpha^2}\oint\bar{\Theta}_{ij}x_j\mathrm{d}x_i\\ &= \mathrm{i}q\oint A_i\mathrm{d}x_i + \frac{\mathrm{i}}{2}q\oint\big[m\theta\cdot(\mathbf{v}\times\nabla A_i)\\ &\quad + q\theta\cdot(\mathbf{A}\times\nabla A_i)\big]\mathrm{d}x_i + \delta\Phi_{\bar{\theta}}^{\mathrm{NCPS}}\,,\end{aligned} \tag{25}$$

where $\delta\Phi_{\bar{\theta}}^{\mathrm{NCPS}}$ is the first-order modification term due to momentum–momentum noncommutativity, and it has the form

$$\begin{aligned}\delta\Phi_{\bar{\theta}}^{\mathrm{NCPS}} &= -\frac{\mathrm{i}}{2\alpha^2}\oint\bar{\Theta}_{ij}x_j\mathrm{d}x_i\\ &\quad + \frac{\mathrm{i}}{2\alpha^2}(1-\alpha^2)q\oint q\theta\cdot(\mathbf{A}\times\nabla A_i)\mathrm{d}x_i\\ &\quad + \frac{\mathrm{i}}{2\alpha^4}(1-\alpha^4)q\oint m\theta\cdot(\mathbf{v}\times\nabla A_i)\mathrm{d}x_i\,.\end{aligned} \tag{26}$$

It is obvious from (26) that, when $\alpha=1$, then we have $\bar{\Theta}_{ij}=0$ as well as $\delta\Phi_{\bar{\theta}}^{\mathrm{NCPS}}=0$, so the AB phase returns to its expression (17) on NC space.

[2] In a similar way as in NC space, we have the relation $m'v_l = p_l - qA_l + O(\Theta) + O(\bar{\Theta})$ on NC phase space, and we omitted the second-order terms of Θ and $\bar{\Theta}$ in (25).

4 Conclusion and remarks

In this article we studied the AB effect in NCQM. The consideration of the NC space (NC phase space) produces an additional phase difference. In order to obtain the NC space correction to the usual AB phase difference, in Sect. 2, first, we gave the Schrödinger equation in the presence of a magnetic field; by solving the equation we derived the magnetic AB phase expression. Note that the noncommutative effects of the space (phase space) in the usual Schrödinger equation can be realized in two steps. The first step is to replace the coordinate and momentum operators with a so-called Bopp's (generalized Bopp's) shift, and then to replace the magnetic potential $\mathbf{A}$ with a special shift which we defined in (10). It is worth mentioning that, on a NC plane, our result (18) coincides with the result of [14]. In order to obtain the NC phase space correction to the usual AB phase difference, in Sect. 3, we solved the Schrödinger equation in the presence of a magnetic field and obtained the magnetic AB phase expression. Especially, the new term $\delta\Phi_{\bar{\theta}}^{\mathrm{NCPS}}$ that comes from the momentum–momentum noncommutativity is given explicitly.

The method we employed in this paper may also be applied to other related physical problems on NC space and NC phase space; for example, the AC effect in NC quantum mechanics. Further study of the related topics will be reported in a forthcoming paper.

Acknowledgements. This paper was completed during our visit to the high energy section of the Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy. We would like to thank Prof. S. Randjbar-Daemi for his kind invitation and warm hospitality during our visit at the ICTP. This work is supported in part by the National Natural Science Foundation of China (90 303 003, 10 575 026 and 10 465 004). The authors are also grateful to the support from the ICTP.

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